Math 250 Proof Portfolio

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Definitions

Definition. An integer x is *even* if $x = 2a$ for some integer a.

Definition. A *rectangle* is a quadrilateral all of whose angles are right angles. A *square* is a rectangle all of whose sides are congruent.

1 Proof by cases

Proposition. If X is a square, then X is a rectangle.

Proof. Suppose X is a square.

I'm going to demonstrate some of the cool ways that LaTeXcan format mathematical expressions! Let s be the side length of X and A be the area of X.

Then we have $A = s^2$. Now suppose that s is an even integer. By definition, this means that $s = 2a$ for some $a \in \mathbb{Z}$. Then the area of X is

$$A = s^2 = (2a)^2 = 4a^2,$$

which we also could have formatted over multiple lines as

$$\begin{aligned} A &= s^2 \\ &= (2a)^2 \\ &= 4a^2, \end{aligned}$$

which shows that A is a multiple of 4, i.e., $A \equiv 0 \pmod 4$.

We can format set-builder notation like this:

$$\{2n : n \in \mathbb{Z}\} \subseteq \{m \in \mathbb{Z} : 4|m\}.$$

Now suppose that the side length of X is a function of t, where $t \in \mathbb{R}$. Then by the chain rule, the derivative of the area is $\frac{dA}{dt} = 2s\frac{ds}{dt}$, which we could also have formatted as $\dfrac{dA}{dt} = 2s\dfrac{ds}{dt}$ if we wanted to make the fractions larger. The net change in the area from $t = t_1$ to $t = t_2$ is

$$A(t_1) - A(t_2) = \int_{t_1}^{t_2} \frac{dA}{dt}\, dt.$$

Notice how I put a period at the end of that sentence!

I said this was a proof by cases, so let's see how to format lists. Either X is small or X is large. Let's consider those two cases separately:

1. Suppose X is small ...

2. Suppose X is large ...

Here's how to create a bulleted list:

- *Case 1:* Suppose X is small ...

- *Case 2:* Suppose X is large ...

By the way, since a square is by definition a rectangle all of whose sides are congruent, X is also a rectangle. This completes the proof. Marvel at how LaTeXwill create the "end-of-proof-box" automatically: □

2 Congruence modulo n

3 Proof by contrapositive

4 Proof by contradiction

5 Induction

6 A proof that a function is bijective

7 The triangle inequality

8 The Pythagorean theorem

Use brackets to give a theorem a name in parentheses:

Theorem (Pythagorean theorem)**.** For every right triangle with legs of lengths a and b and hypotenuse of length c, we have that $a^2 + b^2 = c^2$.

Compare to:

Theorem. This theorem does not have a name in parentheses since I did not use the brackets.

You could cite a source by saying something like: The idea of the following proof, which is originally due to Euclid, has been borrowed from [2].

Proof. Here's the proof. □

9 A proof not from this course

References

[1] R. Hammack, *Book of Proof.* Third edition. 2018.

[2] Wikipedia contributors, *Wikipedia*, "Pythagorean theorem". https://en.wikipedia.org/w/index.php?title= Pythagorean_theorem &oldid=918079404. Online; accessed October 15, 2019.