Murphy Waggoner

Proposition R.231: Prove that $A = \{m + n\sqrt{3} \mid m, n \in \mathbb{Z}\}$ is closed under multiplication.

Proof: Let $A = \{m + n\sqrt{3} \mid m, n \in \mathbb{Z}\}$, and let $m + n\sqrt{3}$ and $p + q\sqrt{3}$ be elements of A. Then

$$(m+n\sqrt{3}) (p+q\sqrt{3}) = mp+mq\sqrt{3}+np\sqrt{3}+3qn = (mp+3qn)+(mq+np)\sqrt{3}.$$

Since $m, n, p, q \in \mathbb{Z}$, mp + 3nq and mq + np are both integers. Therefore,

$$\left(m+n\sqrt{3}\right)\left(p+q\sqrt{3}\right)\in A,$$

and A is closed under multiplication. \Box