Proposition R.231: Prove that $A=\{m+n \sqrt{3} \mid m, n \in \mathbb{Z}\}$ is closed under mulitplication.

Proof: Let $A=\{m+n \sqrt{3} \mid m, n \in \mathbb{Z}\}$, and let $m+n \sqrt{3}$ and $p+q \sqrt{3}$ be elements of A. Then

$$
\begin{aligned}
(m+n \sqrt{3})(p+q \sqrt{3}) & =m p+m q \sqrt{3}+n p \sqrt{3}+3 q n \\
& =(m p+3 q n)+(m q+n p) \sqrt{3} .
\end{aligned}
$$

Since $m, n, p, q \in \mathbb{Z}, m p+3 n q$ and $m q+n p$ are both integers. Therefore,

$$
(m+n \sqrt{3})(p+q \sqrt{3}) \in A
$$

and $A$ is closed under multiplication.

