Runaway Robot II

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Let $\mathbf{x_i}$ be the current position and $\mathbf{d_i}$ and $\mathbf{d_{i-1}}$ be the previous two iterations' motions. Then combine into one state vector:

$$\mathbf{s_i} = \begin{bmatrix} \mathbf{x_i} \\ \mathbf{d_i} \\ \mathbf{d_{i_1}} \end{bmatrix}$$

When moving from $\mathbf{s_i}$ to $\mathbf{s_{i+1}}$, $\mathbf{d_{i+1}}$ can be found by reflecting $\mathbf{d_{i-1}}$ across $\mathbf{d_i}$, while $\mathbf{x_{i+1}}$ is just that step along from $\mathbf{x_i}$:

$$\begin{aligned} \mathbf{d_{i+1}} &= 2\frac{\mathbf{d_i} \cdot \mathbf{d_{i-1}}}{\mathbf{d_i} \cdot \mathbf{d_i}}\mathbf{d_i} - \mathbf{d_{i-1}} \\ \mathbf{x_{i+1}} &= \mathbf{x_i} + \mathbf{d_{i+1}} \\ &= \mathbf{x_i} + 2\frac{\mathbf{d_i} \cdot \mathbf{d_{i-1}}}{\mathbf{d_i} \cdot \mathbf{d_i}}\mathbf{d_i} - \mathbf{d_{i-1}} \end{aligned}$$

Define c_i as the factor for the $\mathbf{d_i}$ term in the above equations:

$$c_i = \frac{2}{\mathbf{d_i} \cdot \mathbf{d_i}}(\mathbf{d_i} \cdot \mathbf{d_{i-1}})$$

Then the update can be linearized across $\mathbf{s}$:

$$F_i = \begin{bmatrix} 1 & 0 & c_i & 0 & -1 & 0 \\ 0 & 1 & 0 & c_i & 0 & -1 \\ 0 & 0 & c_i & 0 & -1 & 0 \\ 0 & 0 & 0 & c_i & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

To use this update matrix with an extended Kalman filter, we need the Jacobian of F_i. As the only non-linearity in F_i, we only need to calculate the partial derivatives of c_i:

$$\frac{\partial c_i}{\partial x_i} = 0$$
$$\frac{\partial c_i}{\partial y_i} = 0$$
$$\frac{\partial c_i}{\partial dx_i} = \frac{2}{\mathbf{d_i} \cdot \mathbf{d_i}} \left[dx_{i-1} - c_i dx_i \right]$$
$$\frac{\partial c_i}{\partial dy_i} = \frac{2}{\mathbf{d_i} \cdot \mathbf{d_i}} \left[dy_{i-1} - c_i dy_i \right]$$
$$\frac{\partial c_i}{\partial dx_{i-1}} = \frac{2}{\mathbf{d_i} \cdot \mathbf{d_i}} dx_i$$
$$\frac{\partial c_i}{\partial dy_{i-1}} = \frac{2}{\mathbf{d_i} \cdot \mathbf{d_i}} dy_i$$

The resulting Jacobian is then:

$$J_i = F_i + \frac{2}{\mathbf{d_i} \cdot \mathbf{d_i}} \begin{bmatrix} dx_i \\ dy_i \\ dx_i \\ dy_i \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ dx_{i-1} - c_i dx_i \\ dy_{i-1} - c_i dy_i \\ dx_i \\ dy_i \end{bmatrix}^T$$

The Kalman filter's prediction step is then:

$$\mathbf{s_{i+1}} = F_i \mathbf{s_i}$$
$$P_{i+1} = J_i P_i J_i^T$$