Determining the Speed of Light

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May 19, 2016

1 Introduction

When measuring a speed, the most common way to calculate it is by recording how far something went and the time it took to go that far. In the case of light, this is very difficult. One could conceivably shine a light over a vast distance and have someone else record when they see the light, but this would be difficult even at large distances. The person recording when they see it will need to have terrific reflexes to accurately measure a correct time as the time will be very short. A better method involves the use of a quickly rotating mirror and a beam of light. By aiming a beam of light off the rotating mirror, then reflecting it off a second stationary mirror back into the rotating mirror, calculations can be made on the speed of light. After first hitting the rotating mirror, the mirror will rotate very slightly in the time it takes the beam of light to return and will reflect back to a different position from where it came from. By measuring the displacement of the round trip, a measurement of the speed of light can be made.

2 Theory

Speed is defined as the distance traveled in a set amount of time, as shown in.

$$s = d/t \tag{1}$$

In this case, the speed we are going to measure is the speed of light, which is denoted as c.

$$c = d/t \tag{2}$$

This is difficult to measure directly, but by studying the path the beam of light takes shown in figure 1 a calculation for the speed of light can be made based off the speed of the rotating mirror.

This then gives a relation between the rotation of the mirror and the distance the light travels when reflecting from the rotating mirror.

$$\omega = \Delta\theta/t \tag{3}$$

Solving (3) for t and plugging it into (2) relates the speed of light to the distance traveled and the speed of the rotating mirror.

$$c = \frac{2d\omega}{\Delta\theta} \tag{4}$$

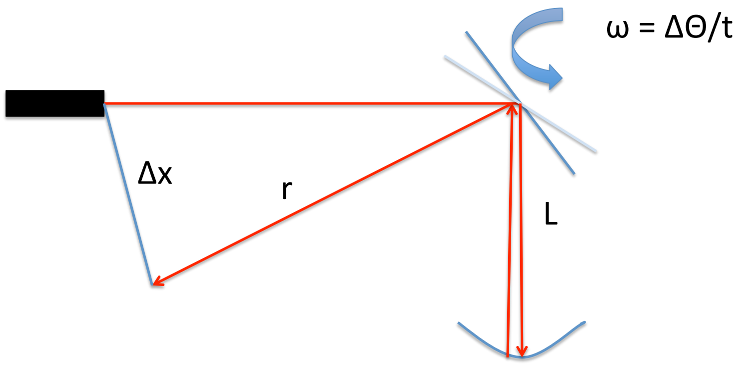

Figure 1: The path taken by a laser beam and the displacement from the rotating mirror.

The distance that will eventually be measured is Δx from 1, and this value is related to the change in angle of the mirror by the following relation.

$$\Delta x = 2r\theta_{beam} \tag{5}$$

An additional factor of two rises in this relation, as the change in angle of the beam is twice the change in the angle of the mirror. This relationship can be plugged into (4), along with substituting frequency, f, for ω, leaving a relationship of measurable parameters.

$$\Delta x = (\frac{8\pi rd}{c})f \tag{6}$$

Reordering the terms for Δx gives an equation where the frequency of the mirror can be set, Δx can be measured, and then the slope of the equation can be determined by a linear curve fit. The slope is equal to everything in the parentheses and this can be used to determine c

3 Set-up and Procedure

When approaching this experimentally in the lab, Δx was expected to very small, so a couple of changes to the basic set up in 1 were needed to measure the small displacement. A microscope was used to measure the small displacement of the returning beam caused by the rotating mirror. This microscope angled a portion of the beam into the eyepiece and let the rest of the light through.

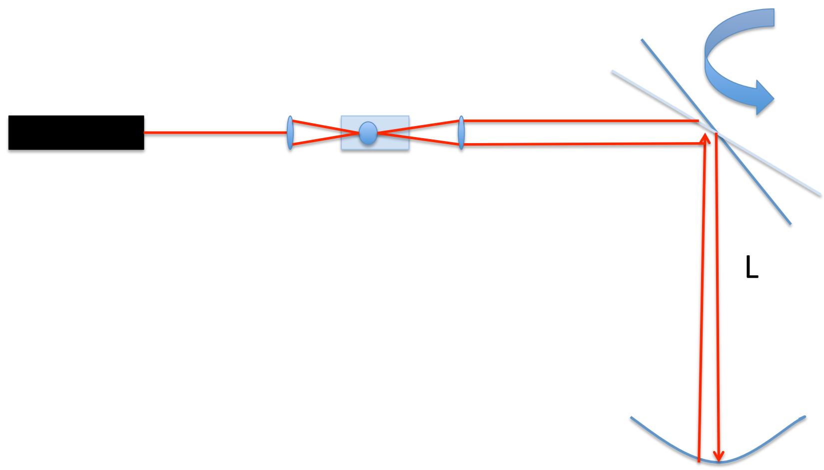

Figure 2: The experimental set up.

In order to accurately determine the displacement, a set of lenses was placed around the microscope in order to focus the beam to a very small point where its position was measured in the microscope. The experimental set up can be seen from figure 2. The length of L is $11.7m$. And the length from the second lens to where the eyepiece measured from was $23.1cm$

4 Data

When measuring data, the frequency of the mirror and the displacement in the microscope were recorded. Several measurements of the mirror spinning in both directions were taken. The data can be seen in figures 3 and 4.

5 Analysis

With the data at hand, the slopes of each linear plot are fitted for, thus a calculation of the speed of light can be made. Setting the items in parentheses from (6) equal to the slopes. Solving for c provides values for the speed of light. When fitting the linear plots, error propagation was added into the data. When measuring the displacement, the microscope had 1mm ticks in its eyepiece. We determined an error of 0.1mm for each measurement of of the displacement.

When solving for c from the slope, the error associated with the microscope is fit into an error propagation model. In this model, the speed of light is solved for 10,000 times with various sizes of the error added or subtracted on by means

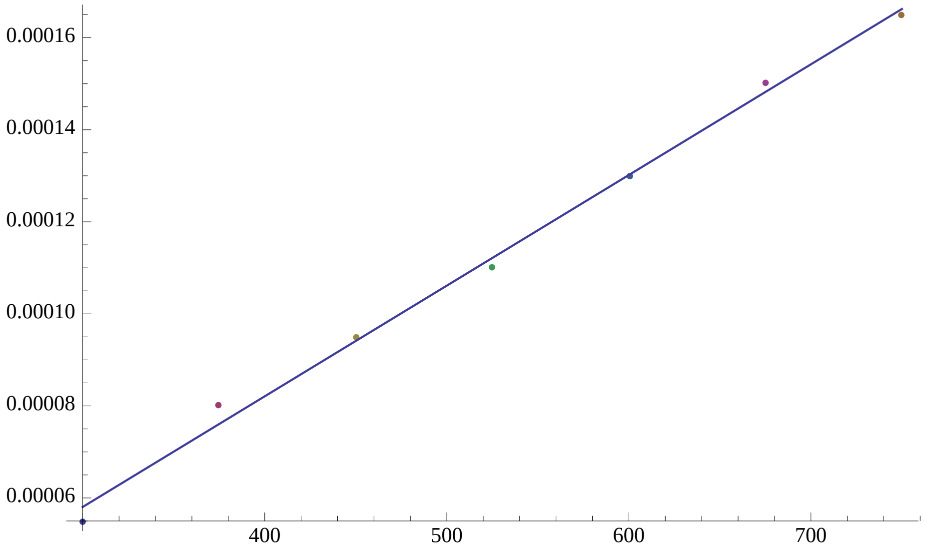

Figure 3: Clockwise data. Slope = $2.40 \times 10^{-7} \pm 6.64 \times 10^{-9}$.

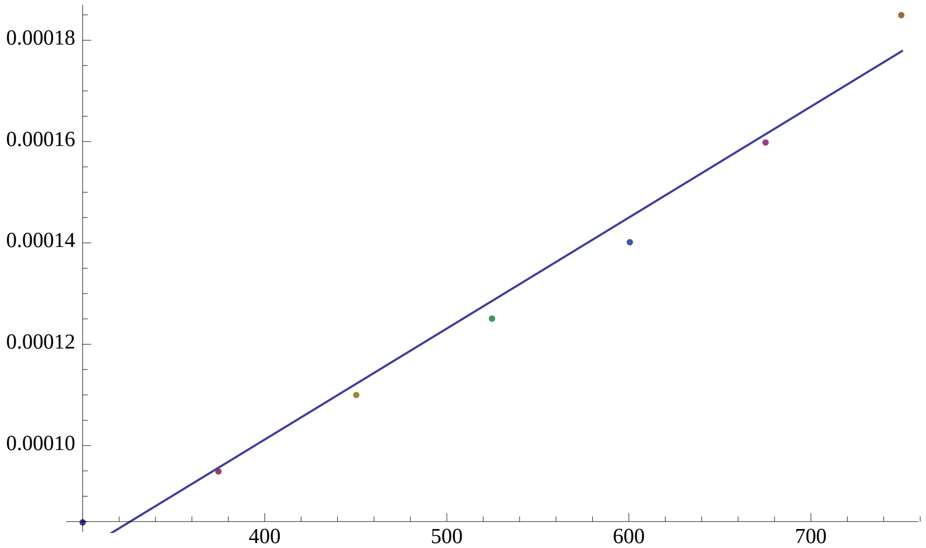

Figure 4: Counterclockwise data. Slope = $2.19 \times 10^{-7} \pm 1.27 \times 10^{-9}$.

of a normal distribution. The mean of this list of values is the calculated speed of light, and the standard deviation of the list is the associated error from the experiment. This command done in Mathematica can be seen in figure 5.

6 Conclusion

Our value determined for the speed of light ranged from $2.82 \times 10^8 \pm 8.14 \times 10^6$ to $3.11 \times 10^8 \pm 1.85 \times 10^7$ which right around the accepted value of c of 2.99×10^8. Our error could have been reduced if we were able to arrange the lenses in a fashion that produced an even sharper point seen in the eyepiece. A sharper point would have allowed for more accurate measurements which would have reduced the error associated with that measurement. Also, the angular speed at which the mirror rotated was also a cause for error, though we determined it less a factor than the eyepiece. Some of our frequency measurements could have had errors as the instrument used did not report the frequency the mirror was spinning at with complete accuracy.

```
In[29]:= speedOfLight = errorprop[ (8 π d f)/slope,
    {{d, 11.7, 0.1}, {slope, 2.404761904761904`*^-7, 6.649638116080472`*^-9}}]
Out[29]= {2.8252 × 10^8, 8.1459 × 10^6}

In[30]:= speedOfLight2 = errorprop[ (8 π d f)/slope,
    {{d, 11.7, 0.1}, {slope, 2.1904761904761904`*^-7, 1.2777531299998782`*^-8}}]
Out[31]= {3.10935 × 10^8, 1.8555 × 10^7}
```


Figure 5: The error propagation commands used in Mathematica